

# Existence of solutions to a pure critical elliptic system in a bounded domain.

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We consider the weakly coupled critical elliptic system

$$\begin{cases} -\Delta u = \mu_1 |u|^{2^*-2} u + \lambda \alpha |u|^{\alpha-2} |v|^\beta u & \text{in } \Omega, \\ -\Delta v = \mu_2 |v|^{2^*-2} v + \lambda \beta |u|^\alpha |v|^{\beta-2} v & \text{in } \Omega, \\ u = v = 0 & \text{on } \partial\Omega, \end{cases} \quad (0.1)$$

where  $\Omega$  is either a bounded smooth domain in  $\mathbb{R}^N$  or  $\Omega = \mathbb{R}^N$ ,  $N \geq 3$ ,  $2^* := \frac{2N}{N-2}$  is the critical Sobolev exponent,  $\mu_1, \mu_2 > 0$ ,  $\lambda \in \mathbb{R}$ ,  $\alpha, \beta > 1$  and  $\alpha + \beta = 2^*$ . This type of systems arises, e.g., in the Hartree-Fock theory for double condensates, that is, Bose-Einstein condensates of two different hyperfine states which overlap in space.

We shall present some existence and multiplicity results for this problem on symmetric domains. This is joint work with Mónica Clapp