Stability of Sine-Gordon 2-solitons in the energy space

José Manuel Palacios *

Abstract

In this talk we will prove that three different 2-soliton solutions of the sine-Gordon equation (SG) are orbitally stable in the natural energy space of the problem [4]. We will prove this result without using the inverse scattering technique for the equation nor the steepest descent method, which allows us to work in the very large energy space $H^1(\mathbb{R}) \times L^2(\mathbb{R})$. The three families which we will study are called 2-kink, kink-antikink and breather of SG, described by Lamb [3]. To prove this result we will use a well-chosen Bäcklund transformation which allow us to reduce the stability question of these families to the zero solution case, in the same spirit as the result of Alejo and Muñoz for the case of the modified Kortweg-de Vries equation [1]. However, we will see that SG presents several new difficulties that we will have to solve appropriately. Possible connections to asymptotic stability results will also be discussed. This work is in colaboration with C. Muñoz and improves in several directions the results in [2].

References

- [1] M.A. Alejo, and C. Muñoz, Dynamics of complex-valued modified KdV solitons with applications to the stability of breathers, Anal. and PDE. 8 (2015), no. 3, 629–674.
- [2] M.A. Alejo, C. Muñoz, and J. M. Palacios, On the Variational Structure of Breather Solutions I: Sine-Gordon case, J. Math. Anal. Appl. Vol.453/2 (2017) pp. 1111–1138. DOI 10.1016/j.jmaa.2017.04.056.
- [3] G.L. Lamb, Elements of Soliton Theory, Pure Appl. Math., Wiley, New York, 1980.
- [4] C. Muñoz, and J.M. Palacios, Stability of the 2-soliton solutions of the SG equation in the energy space, in preparation.

^{*}Departamento de Ingenería Matemática DIM, Universidad de Chile, e-mail: jpalacios@dim.uchile.cl. Parcialmente financiado por Nucleo Milenio CAPDE, CMM Fondo Basal, y Fondecyt 1150202.