TOWERS OF NODAL BUBBLES FOR THE BAHRI-CORON PROBLEM IN PUNCTURED DOMAINS

Let Ω be a smooth bounded domain in \mathbb{R}^N which contains a ball centered at the origin. Consider problem

(0.1)
$$(\wp\delta) \quad \begin{cases} -\Delta u = |u|^{2^* - 2} u & \text{in } \Omega_{\delta}, \\ u = 0 & \text{on } \partial \Omega_{\delta}, \end{cases}$$

here $N \geq 3$, $2^* = \frac{2N}{N-2}$ is the critical Sobolev exponent and $\Omega_{\delta} := \{x \in \Omega : |x| > \delta\}$. In this talk we will discuss the existence of nodal solutions $(u_{m,\delta})$ for problem (\wp_{δ}) . Moreover, if Ω is starshaped, we show that the solutions $(u_{m,\delta})$ concentrate and blowup at 0, as $\delta \to 0$, and their limit profile is a tower of nodal bubbles, i.e., they are a sum of rescaled nonradial sign-changing solutions to the limit problem

(0.2)
$$(\wp_{\delta}) \qquad \left\{ \begin{array}{l} -\Delta u = |u|^{2^{*}-2}u, \quad u \in D^{1,2}(\mathbb{R}^{\mathbb{N}}) \end{array} \right.$$

centered at the origin.

This is a joint work with M. Clapp and F. Pacella.