Fundamental Solutions of Fully Nonlocal Diffusion Problems

Juan Carlos Pozo*
Departamento Matemáticas y Estadística
Universidad de La Frontera
Temuco, Chile

Abstract

The main objective of this work is to analyze temporal decay rates of solutions of a non-local in time and space diffusion problem. Specifically, we study the problem given by the equation

$$\partial_t(\kappa * [u - u_0]) + (-\Delta)^{\beta/2} u = 0, \quad t > 0, \quad x \in \mathbb{R}^N, \tag{1}$$

with initial condition,

$$u(0, x) = u_0(x), \quad x \in \mathbb{R}^N. \tag{2}$$

Here u_0 is a given function and $(\kappa * v)$ denotes the convolution on the positive halfline $\mathbb{R}_+$ with respect to time variable, this is $(\kappa * v)(t) = \int_0^t \kappa(t-s)v(s)ds$, with $t \geqslant 0$. As general hypothesis we assume that κ is a kernel of type $(\mathcal{PC})$, by which we mean that the following condition is satisfied.

($\mathcal{PC}$) $\kappa \in L_{1,loc}(\mathbb{R}_+)$ is nonnegative and nonincreasing, and there exists a kernel $\ell \in L_{1,loc}(\mathbb{R}_+)$ such that $\kappa * \ell = 1$ in $(0, \infty)$. In this case we also write $(\kappa, \ell) \in (\mathcal{PC})$.

Using Prüss and Bochner subordination principles we find fundamental solutions of the problem (1)-(2), and we show how these solutions help to find general solutions. Next, using techniques of Fourier Analysis, we find decay rates of the solution of the problem. We encounter the critical dimension phenomenon where the decay rate attains the decay rate of that in a bounded domain for large enough dimensions. Consequently, the decay rate does not anymore improve when the dimension increases. This work is a generalization of the results obtained in [1].

References

[1] J. Kemppainen, J. Siljander, V. Vergara, R. Zacher, *Decay estimates for time-fractional and other non-local in time subdiffusion equations in* $\mathbb{R}^d$, Math. Ann. 366 (2016), no. 3-4, 941–979.

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