Some results on large-scale dynamics for FitzHugh-Nagumo neurons: the effects of coupling on the fully excitatory case

Cristóbal Quiñinao
Instituto de Ciencias de la Ingeniería - Universidad de O'Higgins

Abstract

In this talk, we present a nonlocal PDEs inspired as the limit of large dynamics of a set of fully connected excitatory FitzHugh-Nagumo (FhN) neurons. This large scale of modelling is useful for understanding the role of mean connectivity and we focus on the study of the long term solutions when the coupling (ε) is either small or large.

In a first part we present the FhN kinetic equation, which appear in the modelling of the evolution of a neuronal network interacting through electrical synapses. Historically, the modelling of these dynamics led to the development of the celebrated Hodgkin-Huxley (HH) model [4], a very precise description of ion exchanges through the membrane and their effects on the cell voltage. Later, the modelling of these biological systems, led to a simplification of the HH model, the Fitzhugh-Nagumo model [3, 6], which has gained the status of canonical model of excitable cells in neuroscience.

In the FhN setting, each neuron is described by two variables: the membrane potential and a recovery function related to the ionic-gates of the cells. When the number N of interacting neurons is large, chaos propagation principle allow us to pass to the limit $N \to \infty$ and describe the system in terms of f_ε, the probability of finding particles in a determined state solving kinetic FhN equation.

In a second part of the talk we present several results we have obtained for this system [5]. In particular we show that in the small connectivity regime, the stationary solution is unique and that nonlinear convergence hols true for this hypo-dissipative equation. Finally, in the a final part of the talk, we present some preliminary results on the system when connectivity is large. Using some classic arguments on Hamilton-Jacobi equations we prove that the Hopf-Cole transformation of f_ε converge to a continuous function φ, such that the limit when $\varepsilon \to \infty$ of f_ε is concentrated around the points where φ touches 0.

This talk is based on [5], a publication done in collaboration with S. Mischler (Université Paris Dauphine), J. Touboul (Collège de France) and some ongoing works done in collaboration with S. Mirrahimi and G. Faye (Université Paul Sabatier).

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