A MIXED FRACTIONAL PROBLEM. MOVING THE BOUNDARY CONDITIONS.

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A natural question when one considers the mixed eigenvalue problem

$$\begin{cases} -\Delta u = \lambda_1(D)u & \text{in } \Omega, \\ u = 0 & \text{in } D, \\ \dfrac{\partial u}{\partial n} = 0 & \text{in } N, \end{cases}$$

where Ω is a Lipschitz bounded domain in $\mathbb{R}^N$, and D, N are submanifolds of $\partial\Omega$ such that

$$\bar{D} \cup \bar{N} = \partial\Omega \quad \text{and} \quad D \cap N = \emptyset,$$

is whether the configuration of the sets D and N determines the behavior of u. Is it similar to the solution of the Dirichlet problem when N is small? or does it behave like the Neumann eigenfunction when N is large? Several authors have shown results where different configurations of D and N provide very different behaviors of u (see for example [1, 2]) depending on the size of the sets, but also on their location.

In this talk we will try to understand the analogous non local problem, that is,

$$\begin{cases} (-\Delta)^s u = \lambda_1(D)u & \text{in } \Omega, \\ u = 0 & \text{in } D, \\ \mathcal{N}_s u = 0 & \text{in } N, \end{cases}$$

where N and D are now two open sets of $\mathbb{R}^N \setminus \Omega$, of positive measure, satisfying

$$D \cap N = \emptyset, \qquad \left|\mathbb{R}^N \setminus (\Omega \cup D \cup N)\right| = 0,$$

and $\mathcal{N}_s u$ denotes the non local Neumann boundary condition of u. As we will see, the fact that the boundary now happens to be the whole $\mathbb{R}^N \setminus \Omega$ instead of $\partial\Omega$ completely changes the possible configurations of the sets (one can even have both sets of unbounded measure).

The purpose of this talk will be to understand what "a small boundary set" means here, and to analyze how D and N can move to recover the classical results.

This is a joint work with T. Leonori, I. Peral, A. Primo and F. Soria, that can be found at https://arxiv.org/pdf/1702.07644.pdf.

References

[1] E. Colorado, I. Peral, *Semilinear elliptic problems with mixed Dirichlet-Neumann boundary conditions.* J. Funct. Anal. 199 (2003), no. 2, 468-507.

[2] J. Denzler, *Windows of given area with minimal heat diffusion*, Trans. of the American Math. Soc. 351 (1999), no. 2, 569–580.